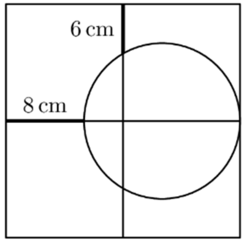
25. Part of the fifth degree polynomial shown cannot be seen because of an inkblot. It is known that all five roots of the polynomial are integers. What is the highest power of $x-1$ that divides the polynomial?

(A) $(x-1)^1$ (B) $(x-1)^2$ (C) $(x-1)^3$ (D) $(x-1)^4$ (E) $(x-1)^5$

26. The large square in the diagram is dissected into four smaller squares. The circle touches the right hand side of the square at its midpoint. What is the side-length of the large square?

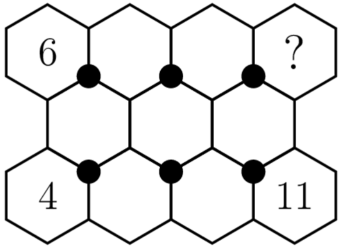


(A) 18 cm (B) 20 cm (C) 24 cm
(D) 28 cm (E) 30 cm

27. What is the smallest positive integer n such that the interval $\left[\frac{n+8}{2}, \frac{2n+14}{3}\right]$ contains at least four natural numbers?

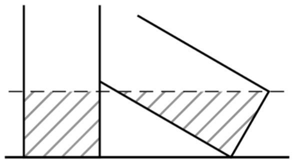
(A) 19 (B) 18 (C) 17 (D) 16 (E) none of the previous

28. The numbers from 1 to 11 are to be placed in the hexagons so that the sum of the three numbers around each of the six black dots is the same. Three of the numbers have already been placed. What number will be placed in the hexagon with a question mark?



(A) 1 (B) 3 (C) 5
(D) 7 (E) 9

29. Two identical cylindrical water tanks contain the same amount of water. One cylinder is standing upright, and the other is leaning against it, and the water level in each of them is the same as in the picture. The bottom of each of the cylinders is a circle with area 3π m^2. How much water, in m^3, does each tank contain?



(A) $3\sqrt{3}\pi$ (B) 6π (C) 9π
(D) $\frac{3\pi}{4}$ (E) it is impossible to determine from the information given

30. The product of six consecutive numbers is a 12-digit number of the form $\overline{abb\,cdd\,cdd\,abb}$, where the digits a, b, c and d are themselves four consecutive numbers in some order. What is the value of the digit d?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Laiks uzdevumu risināšanai – 75 minūtes!

Starptautiskā konkursa „Ķengurs" uzdevumi

23.03.2023. **11.-12. klases**

3 point problems

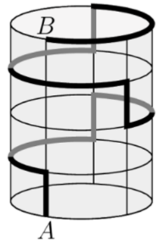
1. What is the value of $\frac{7777^2}{5555 \times 2222}$?

(A) 1 (B) $\frac{7}{10}$ (C) $\frac{49}{10}$ (D) $\frac{77}{110}$ (E) 49

2. Giulia rolls five dice. She rolls 19 points in total. What is the maximum number of sixes she could have rolled?

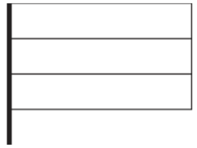
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. A cylindrical can has height 15 cm and the perimeter of its circular base is 30 cm. An ant walks from point A on the base to point B on the roof. Its path is either vertically upwards or horizontally along circular arcs around the can. Its path is shown with a thicker line (black for the path on the front of the can and grey at the back). What is the length, in cm, of the ant's path?



(A) 45 (B) 55 (C) 60 (D) 65 (E) 75

4. Emma has four different coloured pencils. She wants to colour the three-striped rectangular flag shown in the diagram so that each stripe is a single colour and no two adjacent stripes are the same colour. In how many ways can she do this?



(A) 24 (B) 27 (C) 32 (D) 36 (E) 64

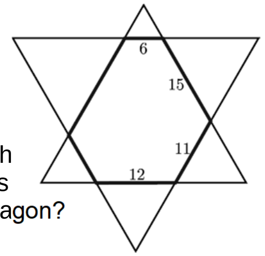
5. We call a positive integer n two-prime, if it has exactly three different divisors, namely 1, 2 and n itself. How many different two-prime integers are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6. How many pairs of positive integers x and y satisfy the equation $x + 2y = 2^{10}$?

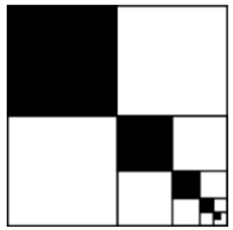
(A) $2^9 - 1$ (B) 2^9 (C) $2^9 + 1$ (D) $2^9 + 2$ (E) 0

7. Two equilateral triangles are put together to form a hexagon with their opposite sides parallel. We know the length of four sides of this hexagon, as shown in the diagram. What is the perimeter of the hexagon?



(A) 64 (B) 66 (C) 68 (D) 70 (E) 72

8. A square with area 84 is divided into four squares. The upper left square is coloured black. The lower right square is again divided into four squares, and so on. The process is repeated an infinite number of times. What is the total area that is coloured black?



(A) 24 (B) 28 (C) 31 (D) 35 (E) 42

9. Each of the integers from 1 to 9 is to be placed in one of the nine boxes in the picture so that any three numbers in consecutive boxes add to a multiple of 3. The numbers 7 and 9 have already been placed. In how many different ways can the remaining boxes be filled?

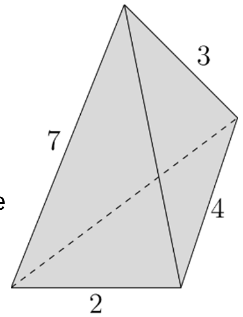
	7		9					

(A) 9 (B) 12 (C) 15 (D) 18 (E) 24

10. What is the units digit of the product $(5^5+1)(5^{10}+1)(5^{15}+1)$?
(A) 1 (B) 2 (C) 4 (D) 5 (E) 6

4 point problems

11. A triangular pyramid has edges of integer length. Four of these lengths are as shown in the diagram. What is the sum of the lengths of the other two edges?



(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

12. For a positive integer n, $n!$ is defined as the product of all integers from 1 to n. For example, $4! = 1\cdot 2\cdot 3\cdot 4 = 24$. What is the sum of the digits of N if $N! = 6!\cdot 7!$?
(A) 1 (B) 2 (C) 4 (D) 8 (E) 9

13. The graphs of the functions $y = x^3 + 3x^2 + ax + 2a + 4$ all pass through the same point, no matter what value of a is chosen. What is the sum of the coordinates of that point?
(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

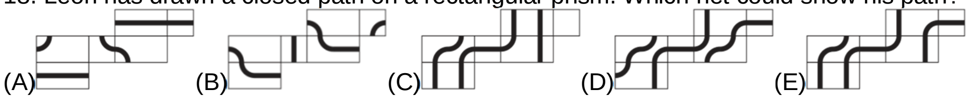
14. We are given five numbers a_1, a_2, a_3, a_4, a_5 whose sum is S. For each integer k, $1 \le k \le 5$, we know that $a_k = k + S$. What is the value of S?
(A) −15 (B) $-\frac{15}{4}$ (C) $-\frac{4}{15}$ (D) $\frac{15}{4}$ (E) 15

15. How many pairs of integers m and n satisfy the inequality $|2m - 2023| + |2n - m| \le 1$?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

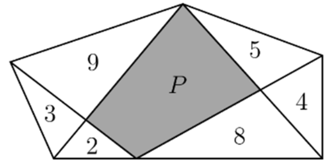
16. There are 23 animals sitting in a row at the cinema. Each animal is either a beaver or a kangaroo. Everyone has at least one neighbour who is a kangaroo. What is the largest possible number of beavers in the row?
(A) 7 (B) 8 (C) 10 (D) 11 (E) 12

17. The number 5^{5^6} can be written in the form n^n for some integer n. What is the value of n?
(A) 5^{30} (B) 5^6 (C) 5^5 (D) 30 (E) 11

18. Leon has drawn a closed path on a rectangular prism. Which net could show his path?



19. A pentagon is dissected into smaller parts, as shown. The numbers inside the triangles indicate their areas. What is the area P of the shaded quadrilateral?



(A) 15 (B) $\frac{31}{2}$ (C) 16
(D) 17 (E) 18

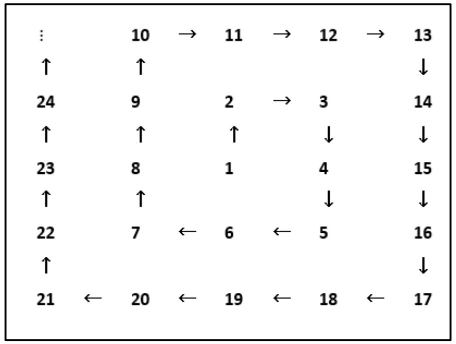
20. How many positive integers are factors of $2^{20}\cdot 3^{23}$ but are not factors of $2^{10}\cdot 3^{20}$?
(A) 13 (B) 30 (C) 273 (D) 460 (E) none of the previous

5 point problems

21. Two functions $f(x)$ and $g(x)$ for any x from the set R satisfy the equalities $f(x) + 2\cdot g(1-x) = x^2$ and $f(1-x) - g(x) = x^2$. What is $f(x)$?
(A) $x^2 - \frac{4}{3}x + \frac{2}{3}$ (B) $x^2 + \frac{4}{3}x + \frac{2}{3}$ (C) $-x^2 - \frac{4}{3}x + \frac{2}{3}$
(D) $x^2 - 4x + 5$ (E) there are no such functions

22. In a bouldering competition, 13 climbers compete in three categories. The score of each competitor is the product of their rankings in the three categories. For example, if one is 4th, 3rd and 6th, their final score is $4\cdot 3\cdot 6 = 72$. The higher your score, the lower your overall ranking. Hannah ranks 1st in two of the categories. What is her lowest possible overall ranking?
(A) 2nd (B) 3rd (C) 4th (D) 5th (E) 6th

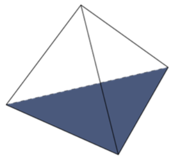
23. A spiral of consecutive numbers is created, as shown, starting with 1. When the pattern of the spiral is continued, in which arrangement will the numbers 625, 626 and 627 appear?

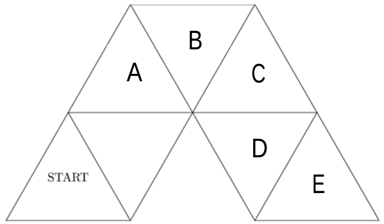


(A) 627 ↑ 626 ↑ 625 (B) 626 → 627, ↑ 625 (C) 625 ↓ 626 ↓ 627

(D) 625 → 626 ↓ 627 (E) 625 → 626 → 627

24. A block in the shape of a regular tetrahedron has one face shaded. The shaded face of the block is placed on the board on the triangle labelled START. The block is then rolled from one triangle to the next by rotating it about one edge. On which triangle will the block stand for the first time again on its shaded face?





(A) A (B) B (C) C (D) D (E) E

11.-12. klases